Eur. Phys. J. A **18**, 189–192 (2003)
DOI 10.1140/epja/i2002-10298-x

THE EUROPEAN PHYSICAL JOURNAL A

The spectrum and strong decays of baryons in a relativistic quark model

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Received: 30 September 2002 /
Published online: 22 October 2003 –

Abstract. On the basis of the three-particle Bethe-Salpeter equation we formulated a relativistic quark model for baryons. With free constituent-quark propagators and instantaneous interaction kernels a good description of the overall baryonic mass spectrum up to the highest spin states is obtained. Preliminary results on strong two-body decays of baryon resonances are discussed.

PACS. 11.10.St Bound and unstable states; Bethe-Salpeter equations – 12.39.Ki Relativistic quark model – 12.40.Yx Hadron mass models and calculations – 13.30.Eg Hadronic decays

1 Introduction

The baryonic resonance spectrum exhibits some striking features: Linear Regge trajectories, which hint at a linear confinement potential; moderately large hyperfine splittings (*e.g.* the N-Δ splitting) hinting at a strong spin-spin interaction; parity doublets, such as *e.g.* $N^*_{\frac{5}{2}^+}(1680)$-$N^*_{\frac{5}{2}^-}(1675)$, which all are a challenge to explain theoretically. The most successful approaches to account for these have been constituent-quark models (in non-relativistic or "relativized" versions), see *e.g.* the excellent review by Capstick and Roberts [1] and references therein, which use one-gluon exchange or Goldstone-boson exchange as quark interaction in addition to a linear confinement potential. Although the results from such calculations are in general satisfactory, they do not reproduce the details of the N Regge trajectory nor explain the parity doublets found. Moreover, the role of the spin-orbit parts of the residual interactions remains obscure. On top of this, the conventional constituent-quark models have no real field-theoretical basis and lack relativistic covariance. As an extension of an earlier relativistic quark model description of mesons [2], we therefore developed a relativistic quark model for baryons on the basis of the three-particle Bethe-Salpeter equation.

2 A relativistic quark model

The details of our approach are extensively described in [3]; here we shall merely quote the basic assumptions and features. Starting point is the Bethe-Salpeter equation for bound states of three fermions, which is a homogeneous integral equation involving full quark propagators and irreducible interaction kernels in terms of the 8 relative momentum variables of the quarks. In order to solve this equation we made the following assumptions, which were inspired by the non-relativistic constituent-quark model being quite successful in describing the baryon spectrum. It is assumed that the self-energy in the quark propagators can be suitably approximated by introducing an effective, constituent-quark mass in the free Feynman propagator. Furthermore, we assume that interaction kernels do not depend on the relative energy variables of the quarks in the rest frame of the baryon. Although this also implies a technical simplification (Salpeter equation), the main reason is that we want to implement confinement by an instantaneous linearly rising three-body potential. These assumptions, after introducing an effective instantaneous kernel that approximates retardation effects in two-body interactions, allow for a formulation of the resulting Salpeter equation as an eigenvalue equation. The latter is solved by expanding the amplitudes in a suitable large, but finite, basis.

Confinement is implemented by a string-like three-body potential, which rises linearly with inter-quark distances and comprises a spin structure chosen such that spin-orbit splittings are suppressed, see [4] for details. In order to account for the hyperfine structure we adopted an effective two-body interaction based on instanton effects, which has the decisive property to solve the $U_A(1)$-problem in the pseudoscalar meson spectrum [2]. For two quarks it is a short-range two-body interaction acting on quark pairs with vanishing spin that are antisymmetric in flavour. Consequently, this force does not act on the flavour symmetric Δ-resonances. The Regge trajectory in

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Table 1. Comparison of experimental (PDG [5]) and calculated masses of resonances on positive-parity Regge trajectories [4].

State	Rating	J^π	PDG	Calc.
$\Delta(1232)$	****	$\frac{3}{2}^+$	1230–1234	1261
$\Delta(1950)$	****	$\frac{7}{2}^+$	1940–1960	1956
$\Delta(2420)$	****	$\frac{11}{2}^+$	2300–2500	2442
$\Delta(2950)$	**	$\frac{15}{2}^+$	2750–3090	2824
$N(939)$	****	$\frac{1}{2}^+$	939	939
$N(1680)$	****	$\frac{5}{2}^+$	1675–1690	1723
$N(2220)$	****	$\frac{9}{2}^+$	2180–2310	2221
$N(2700)$	**	$\frac{13}{2}^+$	2567–3100	2619
$\Lambda(1116)$	****	$\frac{1}{2}^+$	1116	1108
$\Lambda(1820)$	****	$\frac{5}{2}^+$	1815–1825	1834
$\Lambda(1350)$	****	$\frac{9}{2}^+$	2340–2370	2340
Λ		$\frac{13}{2}^+$		2754

this sector is then used to determine the (non-strange) constituent-quark mass and the constant and string tension parameters of the confinement potential. The results are given in table 1. The parameters of the instanton force are determined from the ground-state octet-decuplet splittings. The remaining spectrum is then a genuine prediction.

3 Mass spectra

The resulting mass spectra for non-strange and strange baryons can be found in [4] and [6], respectively. In general a very satisfactory description of the masses of states up to 2.5 GeV is found. The most prominent features are: Once the strengths of the instanton-induced interaction are fixed from the ground-state splittings, see fig. 1, the other prominent hyperfine splittings in the spectrum can be explained quite naturally. In contrast to an earlier calculation in the framework of the non-relativistic quark model with the same interactions [7], in the present relativistic setup the instanton-induced interaction is strong enough to account almost quantitatively for the low position of the Roper resonances and its analogues in the strange sectors. For a more extensive discussion of the interplay of the instanton-induced interaction, the parameterization of confinement and the relativistic treatment, we again refer to [4]. In addition, the N and Λ Regge trajectory can be very nicely reproduced, see also table 1, indicating that this force leads to a constant shift in M^2 for these states, in accordance with experimental data. Moreover this force also accounts for the occurrence of the parity doublets mentioned above: selectively those states from a particular shell (in the harmonic-oscillator classification), which

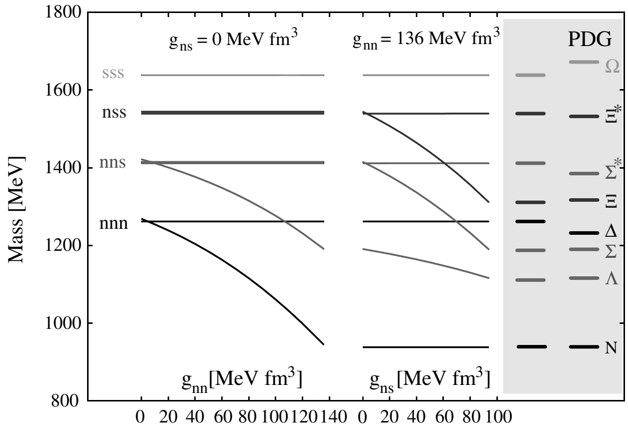

Fig. 1. Octet-decuplet splittings from the instanton-induced interaction. The strengths g_{nn} for non-strange and g_{ns} for non-strange–strange quark pairs increase from left to right, from [4]. Experimental data (PDG) from [5].

show scalar diquark configurations, are lowered enough to become degenerate with some states of the lower oscillator shell with opposite parity [4]. This is illustrated for some higher-spin states in the N-sector in fig. 2. Similar features have also been found for Λ-resonances [4], but not in the other strange sectors, where the instanton-induced force is in general too weak to produce parity doublets. An experimental verification of this would be very interesting.

4 Strong two-body decays

In the framework of the Mandelstam formalism, the amplitude for the strong mesonic decay of excited baryons can be obtained in lowest order by evaluating the simple quark-loop diagram of fig. 3, which involves the vertex functions (amputated Bethe-Salpeter amplitudes) of the participating meson, obtained from a previous calculation on mesons [2], and baryons. A similar procedure has been already used in a treatment of some selected two-meson decays of mainly scalar mesons [8]. Some preliminary results [9] on partial πN- and $\pi\Delta$-decay widths are listed in table 2. In view of the fact that these calculations do not involve any new parameters, we think that the results are quite remarkable. Although the calculated width of the $(\Delta \to N\pi)$-decay is only half the experimental value, we feel that this is quite realistic, since this value leaves room for final-state interaction effects (pion-loop effects), which in general are expected to increase the decay width. Apart from this factor of two we find in general a reasonable description of the partial decay widths of the lowest resonance of given spin-parity. The $N\pi$-decay width of the $N_{\frac{1}{2}^-}(1650)$, $N_{\frac{3}{2}^-}(1700)$ and $N_{\frac{5}{2}^-}(1675)$, which in our calculation are almost pure internal spin-$\frac{3}{2}$ resonances, is definitely too small. At this point we would like to stress that the reconstruction of the nucleon vertex function is approximate only, see [10,11], in contrast to the treatment of Δ-resonances, where no further approximations

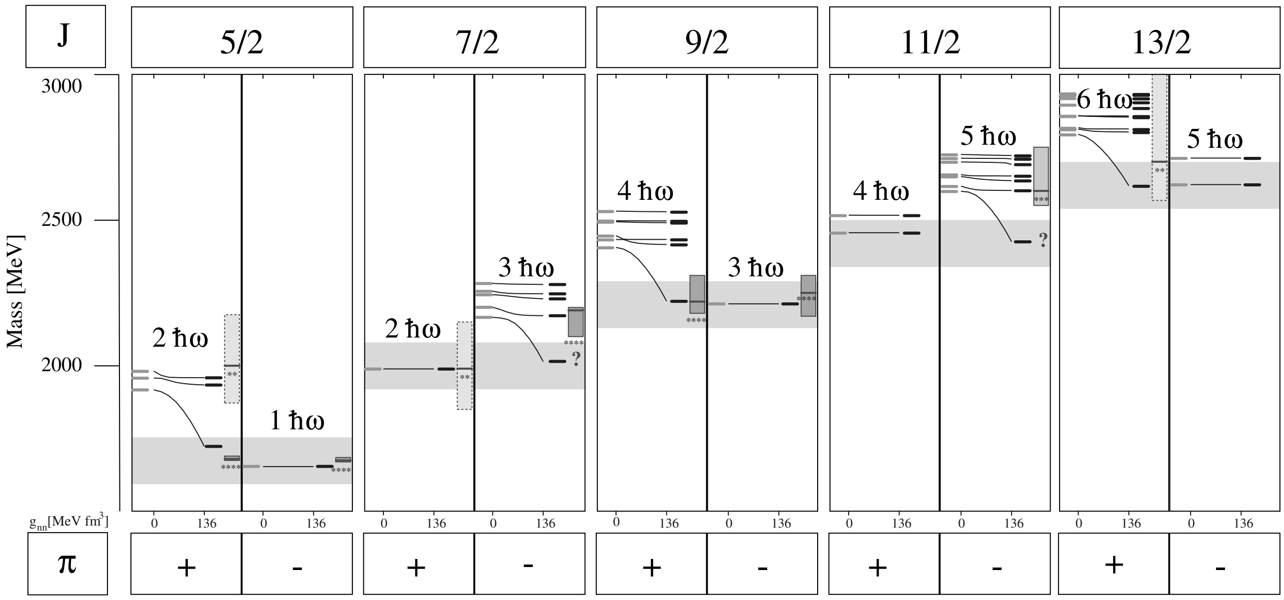

Fig. 2. Generation of parity doublets in the N spectrum through the instanton-induced quark force, from [4]. On the left of each column is the spectrum with the confinement potential alone; the strength of the instanton-induced interaction increases from left to right; experimental data from [5].

Table 2. Comparison of calculated partial decay widths (in MeV) of N- and Δ-resonances to the calculation ($^3P_0$) of Capstick and Roberts [12], and the experimental values (PDG) listed by the Particle Data Group [5].

Decay		Calc.	$^3P_0$	PDG
$P_{33}(1232)$	$\to N\pi$	63	108	$(119 \pm 0)^{+5}_{-5}$
$P_{11}(1440)$	$\to N\pi$	35	412	$(228 \pm 18)^{+65}_{-65}$
	$\to \Delta\pi$	35	11	$(88 \pm 18)^{+25}_{-25}$
$S_{11}(1535)$	$\to N\pi$	34	216	$(68 \pm 15)^{+45}_{-23}$
	$\to \Delta\pi$	1	2	< 2
$S_{11}(1650)$	$\to N\pi$	3	149	$(109 \pm 26)^{+29}_{-4}$
	$\to \Delta\pi$	6	13	$(6 \pm 5)^{+2}_{0}$
$D_{13}(1520)$	$\to N\pi$	39	74	$(66 \pm 6)^{+8}_{-5}$
	$\to \Delta\pi$	35	35	$(24 \pm 6)^{+3}_{-2}$
$D_{13}(1700)$	$\to N\pi$	0.1	34	$(10 \pm 5)^{+5}_{-5}$
	$\to \Delta\pi$	88	778	
$D_{15}(1675)$	$\to N\pi$	4	28	$(68 \pm 7)^{+14}_{-5}$
	$\to \Delta\pi$	35	32	$(83 \pm 7)^{+17}_{-6}$
$S_{31}(1620)$	$\to N\pi$	4	26	$(38 \pm 7)^{+8}_{-8}$
	$\to \Delta\pi$	71	18	$(68 \pm 23)^{+14}_{-14}$
$D_{33}(1700)$	$\to N\pi$	2	24	$(45 \pm 15)^{+15}_{-15}$
	$\to \Delta\pi$	52	262	$(135 \pm 45)^{+45}_{-45}$

are involved and where we get quite reasonable results. Also the issue whether the situation can be improved by taking into account mixings induced by coupling to, *e.g.*, the πN-channel, or that this constitutes a serious deficiency of our internal quark dynamics, will be the subject of future investigations. It can be noted here, that also the photon couplings to these states [10] are underpredicted. For the time being we consider these preliminary results, which, to our knowledge, are the first parameter-free calculations of strong-decay widths in a relativistically covariant framework, as encouraging.

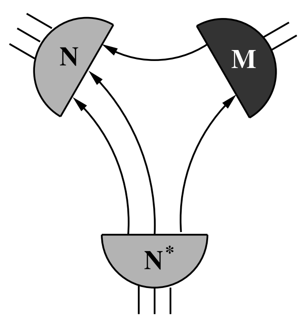

Fig. 3. Lowest-order quark-loop contribution to the strong $(N^* \to N + M)$-decay of excited baryons.

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